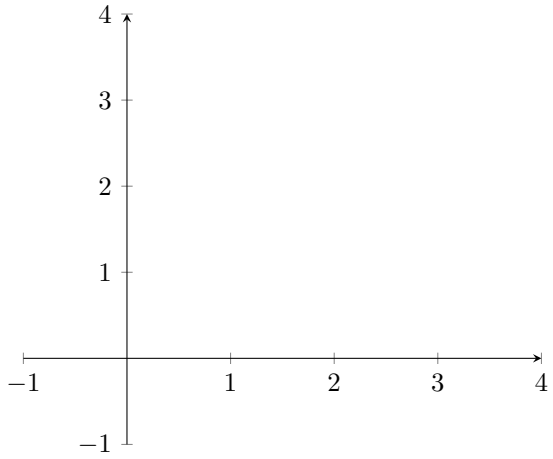
Problem Set 1
Due date: 10/25/17
Total Points: 40 (Weighted ×2.5 for rankings)
If you run out of room for an answer, use scratch paper and staple it to this sheet.

Name and Grade: _____

1. (1 point) Given the following data:

   Graph the data with the horizontal axis being x_1 and the vertical axis x_2. Shade in points that correspond to $y = 0$ and leave hollow the ones that correspond to $y = 1$.

x_1	x_2	y
0	2	0
1	1	0
1	2	0
1	4	1
2	1	0
3	2	1
3	3	1

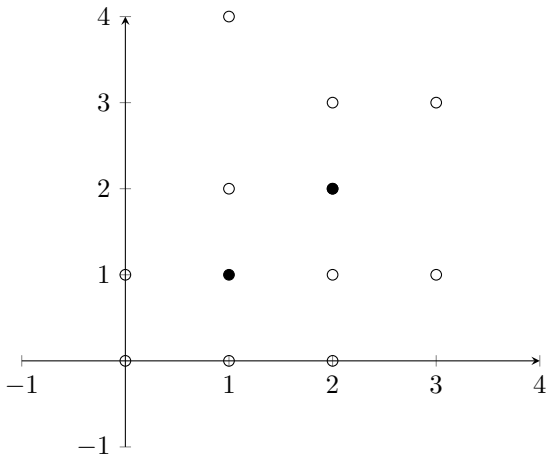


2. (1 point) Draw the decision boundary (line) created by a single perceptron that classifies the data perfectly.

3. (2 points) Write the equation for the decision boundary created above assuming the perceptron outputs 1 when $f(x) > 0$ and 0 when $f(x) < 0$.

4. (8 points) You are now given a perceptron with a decision boundary $x_2 + x_1 - 4 = 0$. Assume the point at (3,2) is moved to (3,1). Use learning rate $\alpha = 0.2$.
   (a) Graph this new decision boundary.
   (b) Update the decision boundary using the misclassified point.
   (c) Was α too high, too low, or fine? (circle one)
   (d) If α was too high or too low, find a value of α that leads to perfect classification after one update step.

5. (10 points) You are given the following data. Hollow points correspond to class 0, shaded points correspond to class 1.



(a) Draw a Multi-Layer Perceptron that perfectly classifies the data above. Make sure to write out the values for every weight and bias in your MLP.

(b) On the graph above, draw the decision boundaries that correspond to each perceptron in your MLP.

6. (10 points) You have a neural network with three layers of nodes. Assume all input values are 1 and all biases are -1. Use the activation function $f(x) = x^2$. Draw a diagram of the network and write the output for each node in the final layer.

$$W_1 = \begin{bmatrix} 3 & 3 & 0 & 1 \\ 1 & 2 & 2 & 3 \\ 3 & 4 & 1 & 5 \\ 1 & 3 & 0 & 2 \end{bmatrix} \qquad W_2 = \begin{bmatrix} 1 & 2 & 2 & 0 \\ 4 & 4 & 2 & 2 \\ 2 & 0 & 1 & 1 \end{bmatrix}$$

7. (3 points) The neural network learning algorithm (backpropagation) will be covered next week. Backpropagation requires a differentiable activation function, which is why neural networks do not use the Perceptron's step function. A popular activation function is the sigmoid function.

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

(a) The sigmoid function is differentiable. Find its derivative.

(b) Rewrite $\sigma'(x)$ in terms of $\sigma(x)$.

8. (2 points) What are the advantages of the Sigmoid function, $\sigma(x) = \frac{1}{1 + e^{-x}}$, over the step function? Circle all that apply.

(a) Sigmoids are differentiable.

(b) $\sigma(x)$ quickly approaches 1 as $x \to \infty$ and $\sigma(x)$ quickly approaches -1 as $x \to -\infty$, so networks learn quickly.

(c) Sigmoids have a simple derivative in terms of the original function, so they are faster to compute than a step function.

(d) The inflection point of $\sigma(x)$ is located on the y-axis, which leads to faster computations with vectorized representations

(e) Sigmoids can tell you how far away you are from the correct value based on the magnitude of x

9. (3 points) There are 16 two-valued boolean functions.

A	B	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	1	0	1	0	1	0	0	1

(a) Which of the boolean functions above cannot be perfectly classified by a single perceptron?

(b) What is/are the name(s) of the function(s) you found in part (a)? (A AND B, A NOT B, etc.)